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Tsunami Models and Velocity of 2004 Indian Ocean Tsunami

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Abstract : There exist many different models to explain tsunamis. These models explain many observed physical facts, but the satellite measured velocity value of 2004 Indian Ocean Tsunami wave cannot be explained by any of the existing models. I here propose a modified KdV model in order to explain the velocity of tsunami wave correctly.

A. Introduction

Tsunami is a Japanese word which means shore wave. In scientific studies, actually it means water waves that start in Deep Ocean due to an underwater earth quake, submarine landslides or volcanoes. People in general are now well aware of tsunamis because of some recent happenings like the Indonesian tsunami of 26th December 2004 that killed more than 200 000 people and 11th March 2011 tsunami of Japan that killed around 15 000 human beings and triggered nuclear mishap. Usually, in case of tsunami, the wavelength is very large about the order of 200km and depth is around 4 or 5kms.This may very well be considered as shallow water disturbances. There are many numerical and analytical attempts to study different tsunamis during last three decades. Here first I want to present a broad picture of the major theoretical studies. Some consider tsunamis as small amplitude shallow water waves which can be described by linear wave equation, some others describe it as N-waves which represent a combination of a solitary elevated pulse and a depressed pulse and still many view it as nonlinear solitonic waves of Kortweg & deVries(KdV) type or Boussinesq type or Kadomtsev & Petviashvili (KP)type.

B. Different Tsunami Models

(i). Linear wave model

Since wave amplitude of tsunami is thousands of times smaller than ocean depth at the place of origin as well as major part of transmission, some believe that it can be described by small amplitude linear waves satisfying linear wave equation ,

$$(\partial^2 u/\partial t^2) - c^2 (\partial^2 u/\partial x^2) = 0, \quad (1)$$

where, u is elevation of water surface from equilibrium value, c is the speed of wave given by $\sqrt{(g\ h)}$; g being acceleration due to gravity and h, the ocean depth. This type of model was used by Arca and Segur [1] to discuss Indian Ocean tsunami of 2004. It at best explains some features like shape very approximately.

(ii). N-Wave model

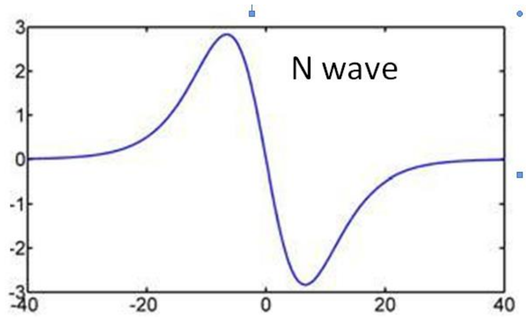
In 1996 Tadepalli and Synolakis [2,3,4] proposed a model to study the receding effect of tsunamis before advancing up the beach. This effect was observed in tsunamis found in Nicaragua, Japan, Indonesia, Russia, Mexico and Peru between 1992and 1996. Usually, solitary waves of sech-type are considered to explain tsunami at the place of generation. In order to explain both receding and advancing nature, Tadepalli and Synolakis proposed the following mathematical model, keeping in view the earth quake of large magnitude:

$$u_{tt} - u_{xx} = h_{0tt}, \quad (2)$$

with $h_0 = (2e_g H/\gamma) \tanh\{\gamma (x - x_1 + ct)\}$,

where h_0 (x ,t) is the ground motion corresponding to a sudden uplift and/or subsidence of the sea bottom due to submarine earth quake, u represents elevation from equilibrium water level, e_g is a scaling parameter defining the crest amplitude ,H gives surface profile and subscripts stand for partial differentiations.

It has N-wave solution having a solitary elevation and depression. For a wave having crest and trough heights equal, the leading elevation propagating along x-axis will be as follows



Their studies claim that most physically realistic tsunamis retain overall N-wave character even after transoceanic propagation. This model explains more features exhibited by tsunamis observed in the recent past. It explains why in some cases water first recedes before advancing in shore lines or in other cases advances without receding ; shape of the wave in transit; nonlinear effects due to quakes in trans oceanic travel, long distance hydrodynamic stability and evolution of waves obeying Green's law. But it fails to explain the satellite measured velocity of the most devastating Boxing Day tsunami of 2004. Some important features in that tsunami as observed are:

(i) It moves in two directions, one with leading elevation wave towards west and the other east ward with a leading depression wave.

(ii) It travels transoceanic distance with average unchanging amplitude just like a soliton.

(iii) Satellite based observations clearly show that it moves with a velocity of 800 km/hr (222 m/s). The Tadepalli and Synolakis model though explains first two observations, it fails to explain the third important fact.

(iii). Boussinesq model

Tsunami waves are usually weakly nonlinear and fairly long water waves and so Boussinesq equation is the better model to deal with such oceanic waves. This equation was first introduced by Joseph Boussinesq in 1872 and many researchers use it to explain various tsunamis in order to include nonlinear and dispersive effects [5].

Taking η as elevation of water surface from equilibrium position, h as the depth of ocean and g as acceleration due to gravity, Boussinesq equation can be written as,

$$\frac{\partial^2\eta}{\partial t^2} - gh\frac{\partial^2\eta}{\partial x^2} - gh\frac{\partial^2}{\partial x^2}\left(\frac{3}{2}\frac{\eta^2}{h} + \frac{1}{3}h^2\frac{\partial^2\eta}{\partial x^2}\right) = 0. \quad (3)$$

The solutions of above equation is

$$\eta = \pm a\, \mathrm{sech}^2 \{½(\sqrt{3a/h^3})[x- ct]\}, \text{ with } c = \sqrt{g(h + a)}$$

This represent left and right moving waves and some workers use its profile [1] to study N-waves.

(iv). Kortweg & de Vries (KdV) model

The most popular model to study tsunami as a nonlinear and dispersive shallow water wave is the Kortweg and de Vries model described by the equation[6],

$$u_t - \alpha\, uu_x + \sigma\, u_{xxx} = 0,$$

where α,σ represent nonlinear and dispersive parameters respectively, u elevations above equilibrium water surface and subscripts stand for partial differentiation. Its solitary wave solutions model tsunami waves which can travel transoceanic distances without change of shape.

(v). Forced KdV model

Pelinovsky and co-workers [7] used forced KdV equation to study atmospheric generated tsunamis, but it can be used to explain different aspects of submarine tsunamis also. The appropriate model equation is given by,

$$u_t - \alpha\, uu_x + \sigma\, u_{xxx} = f_x,$$

where f represents an external perturbing force due to excessive disturbance of water by large magnitude earth quake as in 2004 Indian ocean tsunami.

(vi). Regularized long wave model

Another model used to study [8] tsunamis is based on an improvement in KdV equation and can be written as,

$$u_t + u_x + uu_x - u_{xxt} = 0.$$

First Benjamin, Bona and Mahony proposed it to model surface gravity waves of small amplitude in 1972. It has a solitary wave solution given by

$$u = 3(c^2/1- c^2)\, \mathrm{sech}^2 \{½c\, [x- t / (1-c^2) +\delta]\}.$$ (

For$|c|< 1$, solitary wave has a positive crest elevation and travels in the positive x- direction

(vii).Rivera model

This is a nonlinear dispersive long wave model with the help of which different characteristics of submarine tsunamis are studied numerically by Paul C. Rivera in 2006 [9]. It takes into consideration the effects of earthquake moment magnitude, ocean compressibility through buoyancy frequency, the effects of focal and water depths, and the orientation of ruptured fault line in tsunami magnitude and directivity . The simulations explain a very important observation of the initial receding of water in the east of the source region and the initial flooding in the west coast. It also explains the formation of a series of tsunami waves with periods and lengths as observed using a Fourier series,

$$\eta_i = a_0 + \Sigma_i\, a_i \cos(\omega_i t - \varphi_i)$$

where η_i is a additive constant to the total water depth h, a_0 is the mean amplitude of the tide level and the subscript represent i^{th} tidal constituent.

C. Velocity Study of 2004 Indian Ocean Tsunami

Unlike previous tsunamis, the velocity of the 2004 Indian Ocean tsunami wave was very precisely measured with the use of satellite data [10]. It is found to be 800 km per hour or 222.2 m/s. Almost all models discussed above show that velocity is around $(gh)^{1/2}$ that is 197.07m/s which is nearly 13% less. This prompted me to reconsider the most popular weakly nonlinear and weakly dispersive KdV model. Starting with hydrodynamic Euler equation and using physical boundary conditions at the surface and at the bottom of water D.J.Kortweg & G. de Vries in 1895 proposed an equation which is ubiquitous in its application in numerous fields of physics. In hydrodynamics it appears in phenomena ranging from wake of boat in canals to tsunamis in oceans, but it was first put forth to explain the great wave of translation reported by John Scott Russell in 1844. The form of the equation as derived by Kortweg & de Vries is,

$$u_t + cu_x + (3c/2h)\, u\, u_x + (h^2c/6)\, u_{xxx} = 0,$$

where u is vertical displacement of water surface from equilibrium level, h is the depth of water in rest state, x is distance on water surface along direction of motion of the wave, t is time co-ordinate and subscripts represent partial derivatives.

The above equation possesses a nonlinear wave solution in the form of cnoidal function and the most important solitary scch^2 pulse solution given by,

$$u = A\, \mathrm{sech}^2 \left\{ \frac{1}{2h} \sqrt{\left(\frac{A}{2h}\right)}\, [x - c(1 + A/2h)t] \right\}$$

where, $c = (gh)^{½}$,with g representing acceleration due to gravity

and A is the amplitude .As g is 9.8 m/s^2 and h is 3963 m at the place of occurrence of tsunami ,velocity, v = 197.07 + 0.017 = 197.087 m/s. Since the observed value of the velocity from satellite data is 222.222 m/s, it disagrees with an error of 13%.

Usually equation (9) was intended to explain situations like sudden stoppage of a boat or any other normal disturbances like wind motion. On the other hand tsunami represents a completely different situation where earth quake magnitude, orientation of ruptured fault line, current amplitudes generated during the earth quake, accurate wave height observations and realistic frictional resistance coefficient affects the nonlinearity parameter. Till date no such model exists which takes all these factors in to account and find correct value of velocity as observed in Indian Ocean Tsunami of 2004 [10]. Many have calculated the correct shape of the tsunami wave by introducing extra factors in the equation or in initial conditions. Here I introduce a factor γ in the nonlinearity of the KdV equation and find correct value of the velocity. The modified KdV equation is

$$u_t + cu_x + (3c\gamma /2h)\, u\, u_x + (h^2c/6)u_{xxx} = 0,$$

where γ is designed to take care of the earth quake magnitude, orientation of ruptured fault line, current amplitudes generated during the earth quake, accurate wave height observations and realistic frictional resistance coefficient . The solution of equation(11) is

$$u = A\, \text{sech}^2 \left\{ \frac{1}{2h} \sqrt{\left(\frac{6A\gamma}{2h}\right)} [x - c(1 + A\gamma/2h)t] \right\}$$

with velocity, v = $c(1 + A\gamma/2h)$

if γ = 18X 10^2 , velocity v = 224m/s, with an error of 0.9 %.

Here in choosing γ value unit magnitude of earth quake with all its effects is suggested to contribute a factor of 200 to the nonlinear coefficient.

D. Conclusions

There are different models to explain tsunamis as discussed above. These models explain one or two observed aspects only e.g. linear wave model explains the travel in different directions, the N-wave model explains initial receding of waters of the shore, Boussinesq model demonstrates tsunamis moving in two opposite directions with initial crest or trough etc. So, one accurately measured physical parameter is necessary to know the universality of these models. Since, satellite measured velocity value is very accurate, universality of above mentioned models can be tested by it. As we have seen no model stands the test, I introduce a parameter, the γ factor in the nonlinearity coefficient of KdV equation to explain the velocity value. But this is at best an ad hoc arrangement. It remains to be seen how earth quake magnitude, orientation of ruptured fault line, current amplitudes generated during the earth quake is related to γ .

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